Enrollment No: _____   Exam Seat No: _____

C. U. SHAH UNIVERSITY
Summer Examination-2022

Subject Name: Transform Methods

Subject Code: 4SC05TRM1   **Branch: B.Sc. (Mathematics)**

Semester: 5   **Date: 28/04/2022**   **Time: 11:00 To 02:00**   **Marks: 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**
  a) State Dirichlet's condition for Fourier series.   (02)
  b) State First Shifting Theorem for Laplace transforms.   (02)
  c) Check whether given function is odd or even,   (02)

$$f(x) = \begin{cases} -1, & -\pi < x < -\frac{\pi}{2} \\ 0, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases}.$$

  d) If $F(\lambda)$ is Fourier transform of $f(x)$ then prove that   (02)

$$F\{f(x)\cos ax\} = \frac{1}{2}\{F(\lambda - a) + F(\lambda + a)\}$$

  e) $Z(1) = $ _____.   (01)

   (a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{z}{(z+1)^2}$(d) None

  f) $Z\left(\frac{1}{n!}\right) = $ _____.   (01)

   (a) e^z (b)$e^{\frac{1}{z}}$ (c) e (d) None

  g) $L(\sin at) = $ _____.   (01)

   (a) $\frac{s}{s^2+a^2}$ (b) $\frac{s}{s^2-a^2}$ (c) $\frac{a}{s^2+a^2}$(d) None

  h) $L^{-1}\left\{\frac{1}{\sqrt{s}}\right\} = $ _____.   (01)

   (a) $\frac{1}{\sqrt{t}}$ (b) $\frac{1}{\sqrt{\pi t}}$(c) $\frac{1}{t}$ (d) None

  i) The period of $\sin x$ is_____.   (01)
       (a) π (b) 2π (c) $\frac{\pi}{2}$ (d) None

  j) If $f(x)$ is even function then Fourier co-efficient b_n in Fourier series in   (01)
       the interval $(-l, l)$ is _____.
       (a) 1   (b) -1   (c) 0   (d) None

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**(14)**
a)	State and prove Euler's formula for Fourier series expansion of a function $f(x)$.	(07)
b)	Find the Fourier series of the function $f(x) = \begin{cases} -k & ; -\pi < x < 0 \\ k & ; 0 < x < \pi \end{cases}$ and also deduce that $1 - \left(\frac{1}{3}\right) + \left(\frac{1}{5}\right) - \left(\frac{1}{7}\right) + \cdots = \frac{\pi}{4}$.	(07)

Q-3	**Attempt all questions**	**(14)**
a)	Find the Fourier series of $f(x) = x^2$ in the interval $0 < x < 2$.	(07)
b)	Find the half range cosine series of $f(x) = \pi - x, 0 < x < \pi$.	(05)
c)	Find Laplace transform of $f(t) = t \sin t$.	(02)

Q-4	**Attempt all questions**	**(14)**
a)	Find Fourier transform of the function $f(x) = \begin{cases} 1 & , |x| < 1 \\ 0 & , |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda} d\lambda$.	(07)
b)	Using Fourier integral representation proves that $\int_0^\infty \frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2} d\lambda = \begin{cases} 0 & if x < 0 \\ \frac{\pi}{2} if x = 0 \\ \pi e^{-x} if x > 0 \end{cases}$.	(07)

Q-5	**Attempt all questions**	**(14)**
a)	State and prove Convolution Theorem for Laplace transform.	(06)
b)	Find inverse Laplace transform of $\frac{1}{s(s+1)(s+2)(s+3)}$.	(06)
c)	Find $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$.	(02)

Q-6	**Attempt all questions**	**(14)**
a)	Find the Fourier sine and cosine transform of $f(x) = e^{-ax}, a > 0$.	(05)
b)	Find Laplace transform of the following , i) $e^{-3t}(\cos 4t + 3\sin 4t)$, ii) $2t^3 + e^{-2t} + t^{\frac{4}{3}}$	(05)
c)	If $Z(u_n) = U(z)$ then prove that $Z(u_{n-k}) = z^{-k}U(z)$, where $k > 0$.	(04)

Q-7	**Attempt all questions**	**(14)**
a)	Solve given differential equation by using Laplace transform. $$y'' - 10y' + 9y = 5t, y(0) = -1, y'(0) = 2$$	(07)
b)	Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2. \\ 0, & x > 2 \end{cases}$	(04)
c)	If $F(\lambda)$ is fourier transforme of $f(x)$ then show that $F(f(x-a)) = e^{ia\lambda}F(\lambda)$.	(03)

Q-8   **Attempt all questions**   **(14)**

a) Prove that $Z(n^p) = -Z\frac{d}{dZ}(Z(n^p))$,where p is positive integer.   (05)

b) If $U(z) = \frac{2z^2+3z+12}{(z-1)^4}$ then find the value if u_1 and u_2.   (05)

c) Find $Z(\cos n\theta)$.   (04)